

Loudspeaker Enclosure Design

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Introduction

At low frequencies, where the physical dimensions are small compared to the wavelengths, one can treat the acoustic elements of a loudspeaker enclosure as a lumped parameter system. This model can be combined with the lumped parameter models of the electrical and mechanical components into an equivalent circuit model (see [Appendix A](#)). The net effect is that speakers act as second order high pass filters.

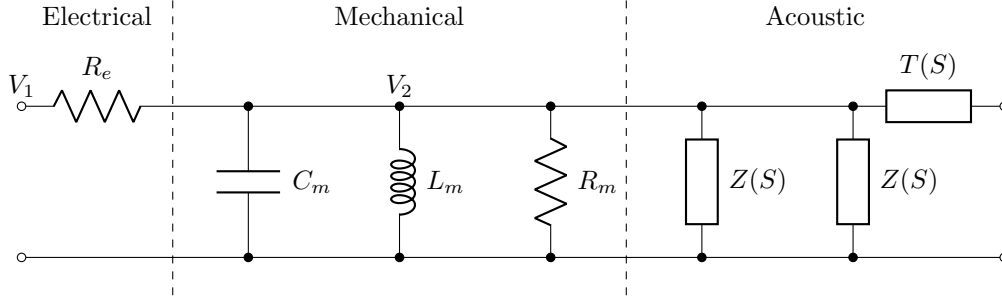
At high frequencies, the acoustics must be modelled as distributed parameter systems where shapes and internal reflections must be taken into account. These effects are largely undesirable and are minimized through the use of sound absorbing materials.

Individual speakers are only useful over a limited range of frequencies. Because of their high pass characteristic, speakers are ineffective below their lumped parameter resonance frequency. Additionally, low frequencies require either greater transit distances or larger surface areas to produce sufficient sound volume. Directionality becomes a problem when the speaker diameter exceeds $1/2$ wavelength. Voice coil impedance can reduce the gain at high frequencies, and the diaphragm has a greater tendency to flex.

The purpose of 2-way and 3-way speakers is to be able to reproduce sound accurately over as much of the audio band (20 Hz to 20 kHz) as possible.

Infinite Baffle

The speaker is mounted into a wall between two infinite half-spaces (or at least two large rooms). The equivalent circuit for this speaker is shown below.

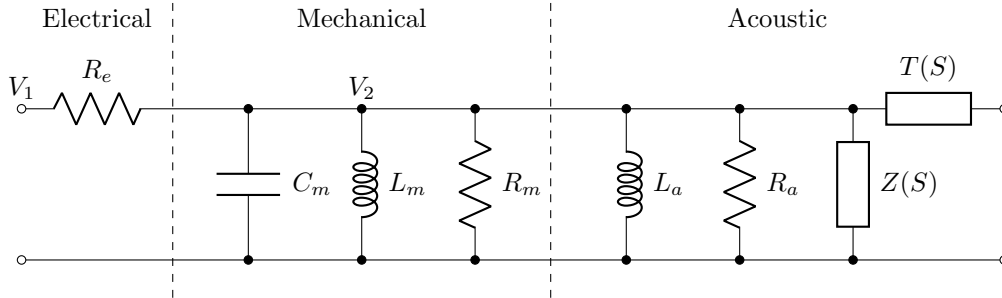


R_e is the DC electrical resistance (which can be measured directly), C_m is proportional to the piston mass, L_m is proportional to the mechanical compliance and R_m is inversely proportional to mechanical resistance. $Z(S)$ and $T(S)$ represent the load and coupling between the diaphragm and output sound for infinite half-spaces (see [Appendix E](#)). $Z(S)$ applies to both sides of the speaker in this case. What is important is that $Z(S)$ is very large and can be ignored, while $T(S)$ is proportional to frequency, turning what would otherwise be a simple resonator into a high pass filter.

The transfer function and input impedance for this model are given by equations (B3) and (B4), after substituting $R_1 = R_e$ etc. Real world examples are given in [Example 1](#) and [Example 2](#).

Simple Enclosure

The equivalent circuit model for a simple speaker enclosure is shown below.



R_e , C_m , L_m and R_m are characteristics of the speaker itself. L_a is proportional to the volume of the enclosure and R_a is inversely proportional to the amount of sound absorbing material used.

Note, L_m and L_a can be replaced by a single inductor

$$L = \frac{L_m L_a}{L_m + L_a}$$

which is smaller than either L_m or L_a individually. R_m and R_a can also be replaced by a single resistor

$$R = \frac{R_m R_a}{R_m + R_a} .$$

Consequently, the analysis of the simple enclosure and the infinite baffle are almost identical.

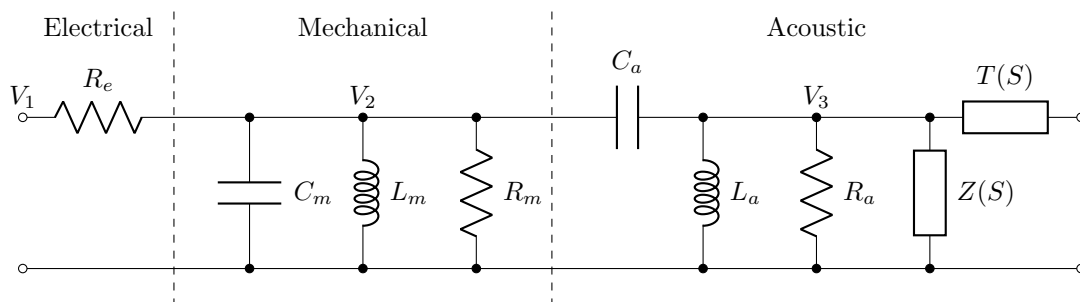
The transfer function and input impedance for this model are given by equations (B5) and (B6), after substituting $R_1 = R_e$ etc. Real world examples are given in [Example 1](#) and [Example 2](#).

Typically one would like to have a maximally smooth (Butterworth) response with as low a resonance frequency as possible. Increasing the mass of the speaker diaphragm will lower the resonance frequency, but also reduces passband gain. Increasing the volume of the enclosure also will lower the resonance frequency, but will never reduce it below its unenclosed (infinite baffle) value. Adding sound absorbing materials will increase damping. In the unlikely event that the speaker is over damped, one could reduce the damping by adding a resistor in series with the speaker.

Bass Reflex

A bass reflex or vented speaker uses a “tuned port” to vent some of the sound from the enclosure. This has little effect on higher frequencies but causes an additional 6 dB/octave roll off below the resonance frequency. The idea is to provide additional damping at the resonance frequency, especially since sound absorbing materials do not work well at lower frequencies.

The equivalent circuit for a bass reflex speaker is shown below.



C_a is proportional to the length of the tube divided by its area. (see [Appendix A](#)). The tube and enclosure cavity appear in series since they have a common pressure (through variable) and the total mass flow (across variable) equals the sum of the mass flows for the two components. The sound pressure at a distance is proportional to the mass flow from the speaker minus the mass flow from the port, which equals the mass flow across L_a (hence the placement of $Z(S)$ and $T(S)$).

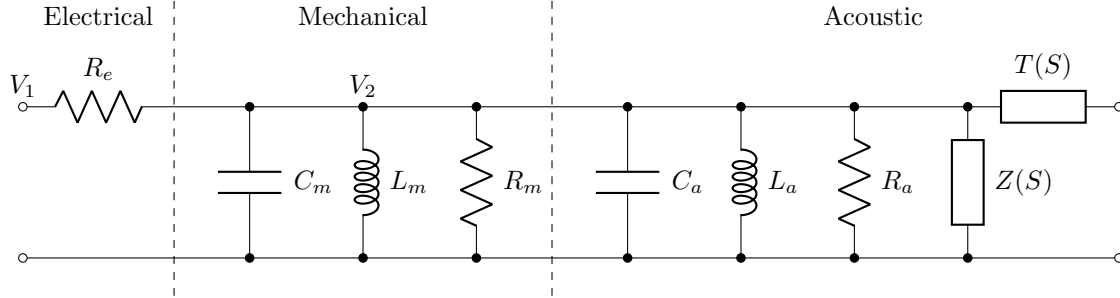
A more rigorous treatment would take into account the sizes of the two different ports while computing air loading. However, their effects over the frequencies of interest are negligible.

The transfer function and input impedance are given by equations (B7) and (B8), after substituting $R_1 = R_e$ etc. The traditional solution is to set $C_a L_a = C_m L_m$ and adjust R_a to reduce the peak gain. Alternatively, one can produce a fourth order Butterworth response for specific values for R_a , L_a and C_a for a given speaker (R_e , R_m , L_m and C_m) as shown in [Example 1](#).

Omni-directional Speakers

One solution to the directionality problem is to aim the speaker directly upward or downward and reflect the sound off a 45 degree cone. There is still vertical directionality, but less so than for the original (a 5 inch disk is imaged as a 2.5 inch line source) and much less so than for a corresponding tower speaker system. Besides, ears don't move up and down that much.

The woofer doesn't need a reflector, since the wavelengths produced are greater than the physical dimensions over the entire pass band. One can instead simply aim the speaker down at the floor or up at a flat barrier. The equivalent circuit for an omni-directional speaker is shown below. One basically treats the space between the speaker and the surface of the cylinder formed by the barrier as a tuned port.



C_a can be approximated using the volume of air inside the cylinder divided by the product of the surface areas of the speaker and the cylindrical opening. C_m and C_a can also be replaced by a single capacitor

$$C = C_m + C_a \quad .$$

Consequently, the analysis of the omni-directional speaker and the simple enclosure are almost identical. When not using a conical reflector, $T(S)$ will be more frequency dependent (worst case directionality).

Enclosure Shapes

Any enclosure has certain frequencies at which it resonates due to standing waves. Energy is stored when the speaker drives those frequencies, then released again causing the speaker to “hum.” Sound absorbing materials reduce the problem, as does mechanical or electrical resistance. It really doesn’t matter where the absorption comes from in terms of effectiveness.

Since the effect of sound absorption is to lower and broaden the “spikes” in the frequency response, they work better with many weak resonances than with a few strong ones. That is why one chooses relatively prime dimensions for rectangular boxes.

For low frequencies, enclosures need a lot of volume, but shapes with good volume to surface area ratios have horrible resonance characteristics. Most enclosures consist of rectangular boxes which have good volume to surface area ratios and are **packed** with sound absorbing materials.

Ideally one would like an enclosure to have all of the standing wave resonances outside the pass band for the speaker. One such solution is to combine two short dimensions and one very long dimension (a wave guide or labyrinth).

Shapes with sloped walls do better than rectangular boxes and prisms with a regular polygonal base. It should be noted that enclosures with sloped walls can be treated as horns (see [Appendix G](#)). For a horn where the area goes to zero (cones, pyramids and wedges), one inevitably enters a region where the phase velocity goes to infinity and the amplitude decreases, yielding no reflection. The lower the frequency, the larger this region becomes.

Crossover Filters

The primary goal of crossover filters is to combine the speakers of a 2-way or 3-way system so as to provide accurate reproduction over the entire range of frequencies. The second goal is to reduce directionality by switching quickly to the smaller speaker. The third goal is to avoid driving speakers below their operating range, which is both a waste of power and a danger to the speaker. Lastly, voltage dividers may be needed to equalize the sound efficiency of the different speakers, or one can use an active filter design where each speaker gets its own driver (powered speakers).

A combination of a fourth order low-pass filter and a fourth order high-pass combined filter/speaker with matching poles will have a Laplace transform of the form

$$F(S) = \frac{S^4 + \omega_c^4}{S^4 + a_1\omega_c S^3 + a_2\omega_c^2 S^2 + a_3\omega_c^3 S + \omega_c^4}$$

where $\omega_c = 2\pi f_c$ is the cutoff frequency and the coefficients a_1 , a_2 and a_3 are yet to be determined. For simplicity, assume $\omega_c = 1$.

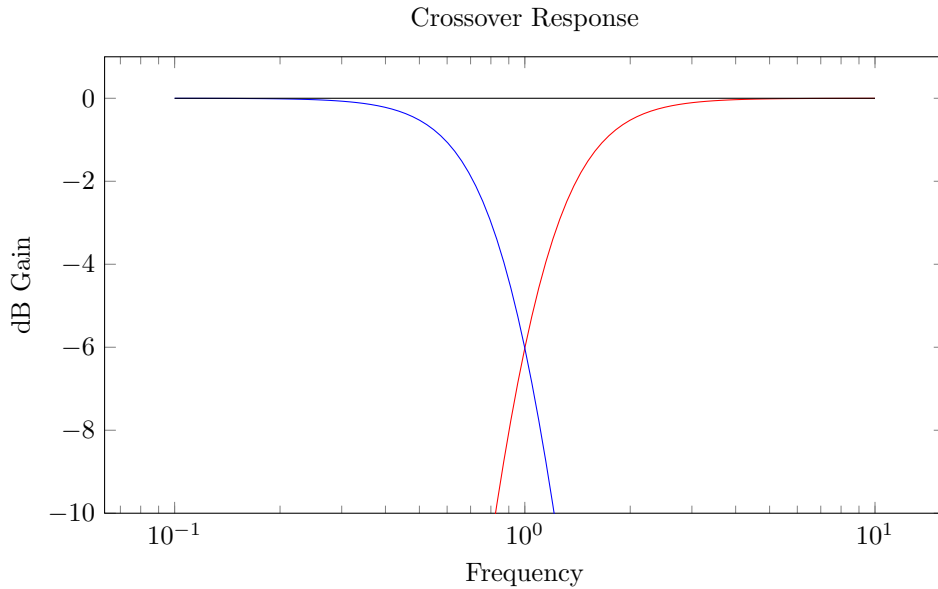
One can show that

$$S^4 + 1 = (S^2 + \sqrt{2}S + 1)(S^2 - \sqrt{2}S + 1)$$

which does not have a stable inverse. However,

$$F(S) = \frac{S^4 + 1}{(S^2 + \sqrt{2}S + 1)^2}$$

has a perfectly flat response, as shown below. This corresponds to a double second order Butterworth filter (which is not the same as a fourth order Butterworth).



Compensation Filters

One can achieve any desired speaker response by over driving the low frequencies to compensate for the limitations of the speaker itself. However, there are a few caveats.

Lower frequencies require longer traverses of the diaphragm to achieve the same velocity and therefore the same sound volume. A speaker diaphragm will only travel so far before it starts to hit things. A given

speaker therefore has a natural limit on how much bass it can produce without distortion. Increasing the diaphragm size increases the amount of bass which can be produced, but also requires a larger enclosure (L_a is multiplied by the diaphragm area squared).

Attempting to drive a speaker below its resonance frequency requires a lot of power, possibly exceeding the amplifier's capability or the speaker's ability to dissipate heat.

The speaker from **Example 1** with a 20 liter enclosure (too small) has a normalized response of

$$V(S) = \frac{S^2}{S^2 + 417S + 134230} \quad .$$

One can use the boost function

$$B(S) = \frac{S^2(S^2 + 417S + 134230)}{S^4 + 328S^3 + 53844S^2 + 5.18 \times 10^6S + 2.49 \times 10^8}$$

to effect a fourth order Butterworth response with a 20 Hz cutoff frequency. In the following graph, the blue line shows the normal speaker response, the red line shows the boosted signal $B(S)$ and the black line shows the boosted speaker output $B(S)V(S)$. Note that the filter also corrected for the effect of the smaller enclosure.

