Imaginary and Complex Numbers

Going beyond “all reals”

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- An entire branch of calculus was developed using line integrals in the complex plane.

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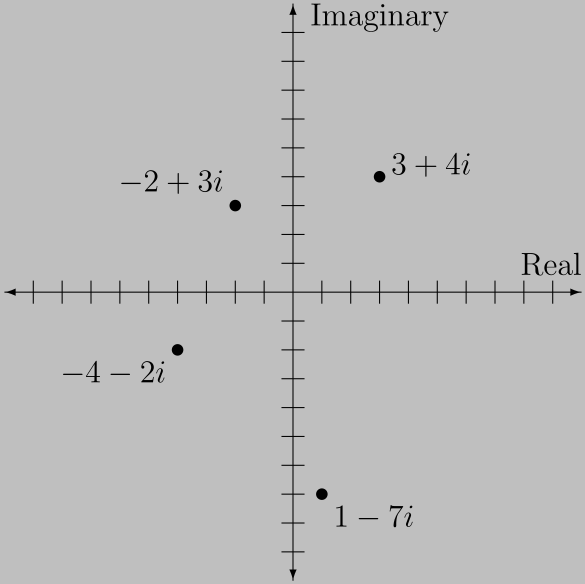
Complex numbers have both real and imaginary parts and can be simplified to the form

$$a + bi \quad \forall a, b \in \Re$$

Complex Plane



Powers of i

$$
\begin{aligned}
i^0 &= 1 \\
i^1 &= i = \sqrt{-1} \\
i^2 &= -1 \\
i^3 &= -i \\
i^4 &= -i^2 = 1 \\
i^5 &= i \\
&\vdots
\end{aligned}
$$

$$i^{-1} = \frac{1}{i} = \frac{i}{i^2} = -i$$

Complex Addition

Real and imaginary terms are added separately:

$$(a + bi) + (c + di) = (a + c) + (b + d)i$$

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Complex Multiplication

Multiply complex numbers like binomials:

$$\begin{aligned}(a+bi)(c+di) &= ac + adi + bci + bdi^2 \\ &= (ac - bd) + (ad + bc)i\end{aligned}$$

since $i^2 = -1$.

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Complex Conjugates

Changing the sign of the imaginary component gives the complex conjugate.

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Most useful property:

$$\begin{aligned} (a + bi)(a - bi) &= a^2 - abi + abi - bi^2 \\ &= a^2 + b^2 \end{aligned}$$

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Complex Absolute Value:

$$|x| = \sqrt{x^* x}$$

Distance to origin of complex plane.

Complex Division

$$\begin{aligned}\frac{a+bi}{c+di} &= \frac{a+bi}{c+di}\left(\frac{c-di}{c-di}\right)\\ &= \frac{(a+bi)(c-di)}{c^2+d^2}\end{aligned}$$

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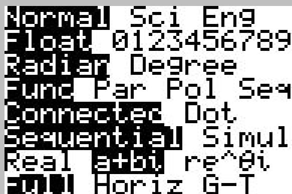
$$\left(\frac{11+2i}{25}\right)\left(\frac{11-2i}{5}\right) = \frac{121+4}{125} = 1$$

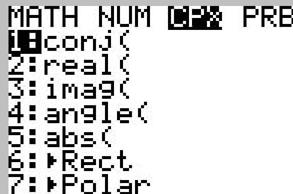
Complex Numbers on the Calculator

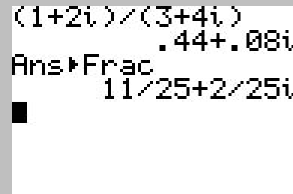
Under the [MODE] menu you will find the [a+bi] option.

The [MATH][CPX] menu includes functions specific to complex numbers.

The [2nd][.] key produces $i = \sqrt{-1}$.







But that takes all the fun out of it!