Introduction to Algebra

Al-jabr is Arabic for restoration.

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Symbolic algebra reached full maturity with the publication of Descartes' **La Géométrie** in 1637.

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The division operator, $\div$, is replaced by / or fractions.

$$x/2 = \frac{x}{2} = \frac{1}{2}x$$

Algebra in Science and Engineering

Relativity

$$E = mc^2$$

where $c \approx 3 \times 10^8$ m/sec.

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Ohm's Law

$$V = IR$$

Linear Algebra

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where a and b are constants and x is the variable.

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In all of these, $a = 0$ represents a special case.

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How does one simplify?

Algebraic Properties

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These are true for all possible values of a, b, c, and x.

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Does it work for division?

$$a/b \neq b/a \quad \text{for all} \quad a \neq b$$

Division can be treated as multiplication by reciprocals.

$$a/b = a\left(\frac{1}{b}\right) = \left(\frac{1}{b}\right)a$$

Re-ordering Operations

$$\begin{aligned}
a + b + c &= (a + b) + c & \\
&= (b + a) + c & \text{commutative} \\
&= b + (a + c) & \text{associative} \\
&= b + (c + a) & \text{commutative} \\
&= (b + c) + a & \text{associative} \\
&= (c + b) + a & \text{commutative} \\
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When dealing only with additions, the order doesn't matter.

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\begin{aligned}
abc &= (ab)c & \\
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&= b(ac) & \text{associative} \\
&= b(ca) & \text{commutative} \\
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When dealing only with multiplication, the order doesn't matter.

When mixing additions and multiplication, order **does** matter.

Simplifying Expressions

(1) distribute multiplications and sign changes.

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Collect x terms:

$$2x - 7$$

Linear expression:

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Collect x terms:

$$2x^2-x+1$$

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Distribte x into $(2x+3)$:

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Distribute -4 into $(x-1)$:

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Quadratic expression:

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