Euclidean Geometry

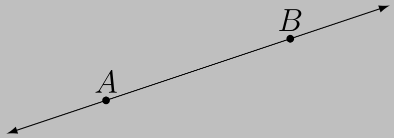
Introduction

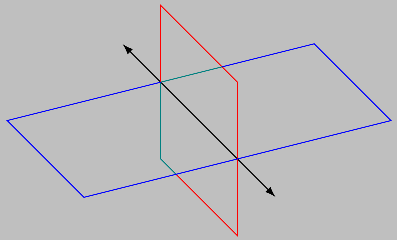
Undefined Terms

A point is like a dot, only smaller. It has a location but no size.

A line is like a drawn line, only thinner, straighter and longer.
It extends through all space along a specific direction but has no width.
The shortest path between any two points is along straight line.

A plane is like a flat surface, only thinner, flatter and bigger.
It extends through all space in more than one direction but has no thickness.





Points, lines and planes are used to construct all the Defined Terms.

Definitions

Collinear Points all lie along one line.

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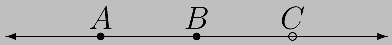

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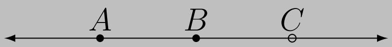

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Points are named using single upper case letters: P

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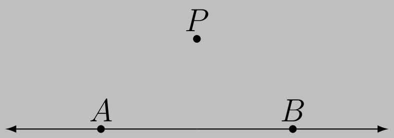

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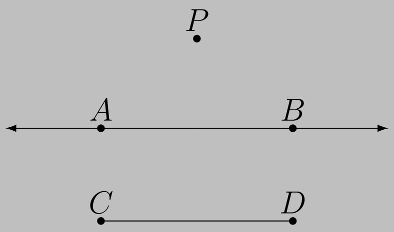

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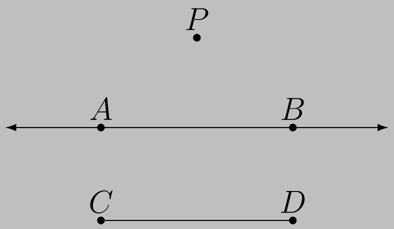

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One can also assign labels, as in line ℓ or plane p.

Distance

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$$AB + BC = AC$$

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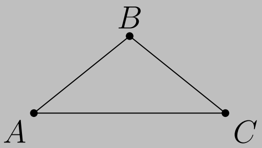
If points A, B and C are non-collinear, then

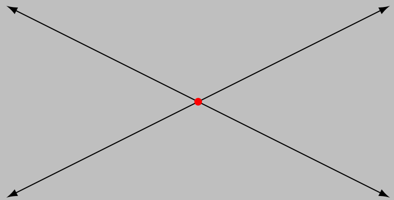
$$AB + BC > AC$$

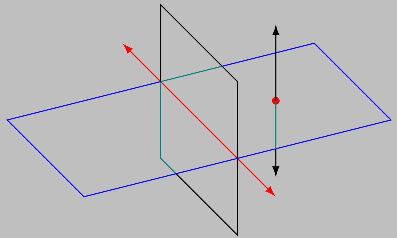


Intersections

Two geometric figures intersect if they have one or more points in common.

Two lines intersect at a point.
Two planes intersect at a line.
A plane and a non-coplanar line intersect at a point.





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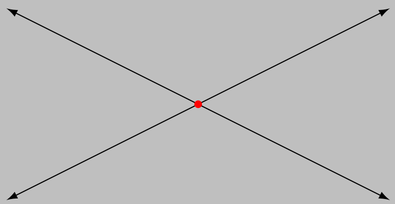
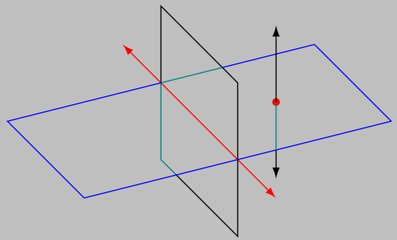
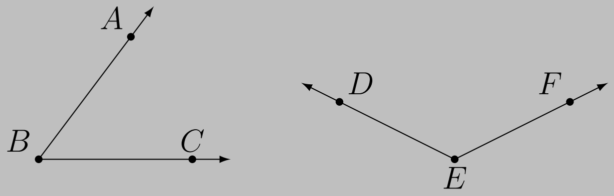




Parallel planes do not intersect.
Parallel lines are coplanar and do not intersect.
Skew lines are non-coplanar (can not intersect).

Angles

An angle consists of 2 rays (sides) with a common endpoint (vertex).
The difference in the directions of these rays is the measure of the angle.



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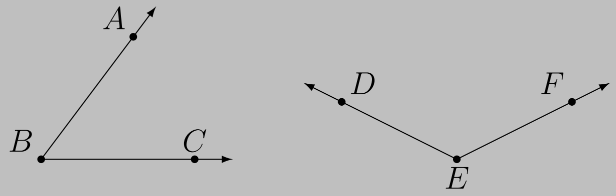

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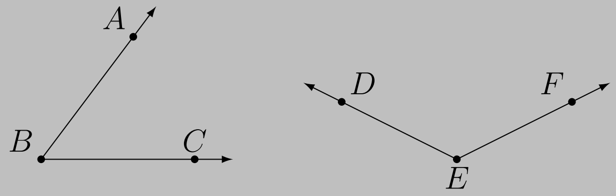

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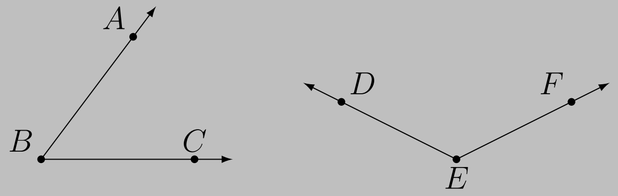

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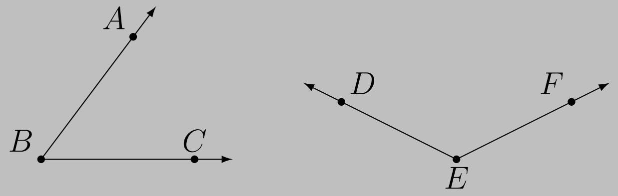

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Note: When we say "angle" we usually mean "the measure of an angle." An angle is a geometric figure, not a number.

Congruent

Numbers are equal. Geometric figures are congruent.

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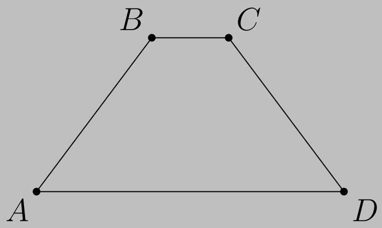
Line segments are congruent if their measures (lengths) are equal.

$$\overline{AB} \cong \overline{CD} \quad \text{if} \quad AB = CD$$

Angles are congruent if their measures are equal.

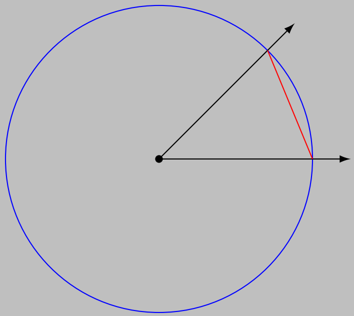
$$\angle A \cong \angle D \quad \text{if} \quad m\angle A = m\angle D$$



Measuring Angles

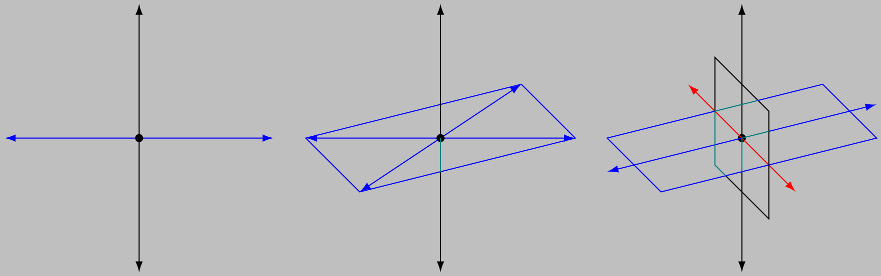
We measure angles with protractors using the Babylonian system of degrees ($360°$ in a circle). In geometry, angles are always positive and less than or equal to $180°$.

Euclid measured angles by drawing a circle and measuring the distance between the points where the circle intersects the rays.



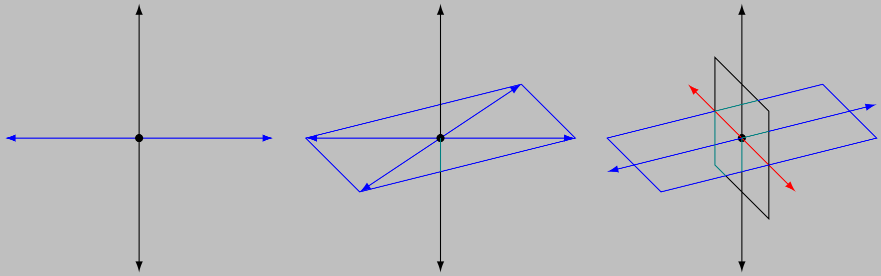
This will tell you when angles are congruent, larger or smaller, but not much else.

Perpendicular



Two rays (with a common endpoint) are perpendicular if the angles formed with one of the rays and its opposite ray are congruent.

Perpendicular



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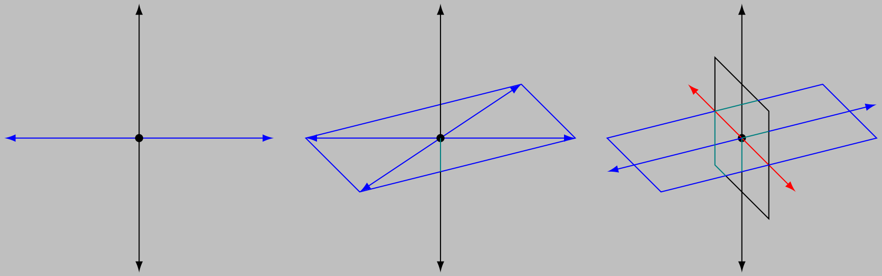

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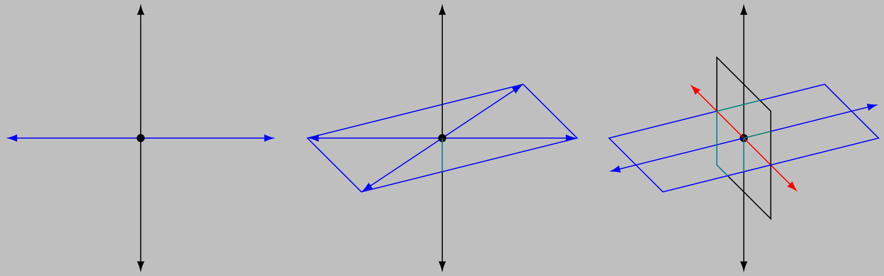

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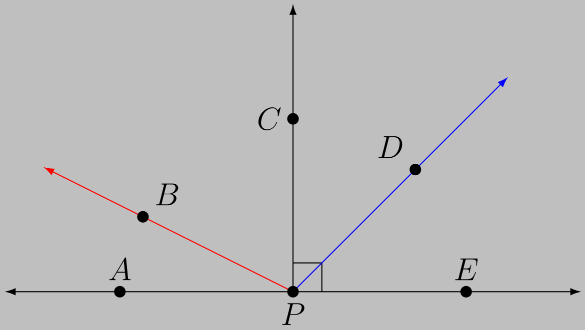
Two planes are perpendicular if one plane contains a line perpendicular to the other plane.

Named Angles



A Straight Angle is formed by opposite rays (180°): $\angle APE$

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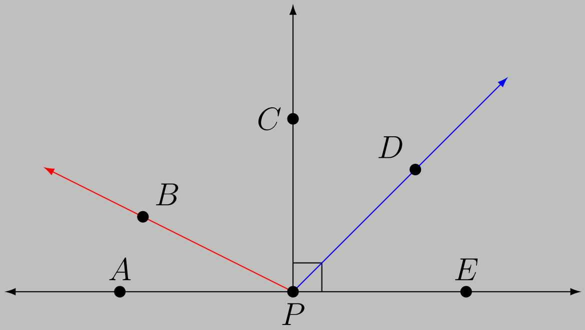

A Straight Angle is formed by opposite rays (180°): $\angle APE$

A Right Angle is formed by perpendicular rays (90°): $\angle APC$ and $\angle CPE$

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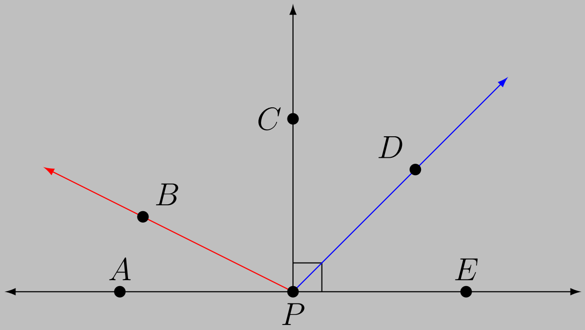

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An Acute Angle has a smaller measure than a right angle (between 0° and 90°): $\angle APB$, $\angle BPC$, $\angle CPD$ and $\angle DPE$

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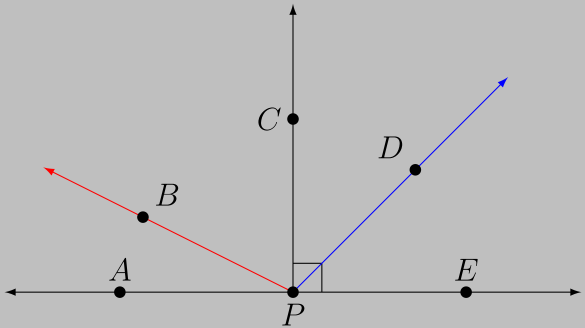

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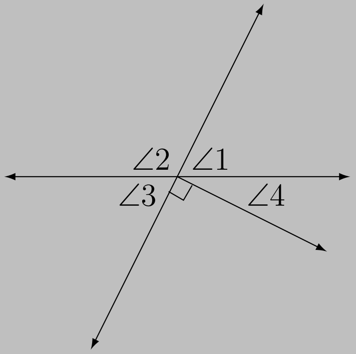
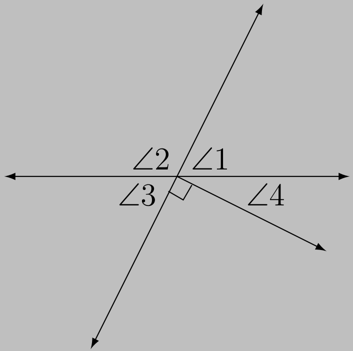
An Obtuse Angle has a larger measure than a right angle (between 90° and 180°): $\angle APD$, $\angle BPD$ and $\angle BPE$

Pairs of Angles



Adjacent Angles - Two angles with a common side (ray).

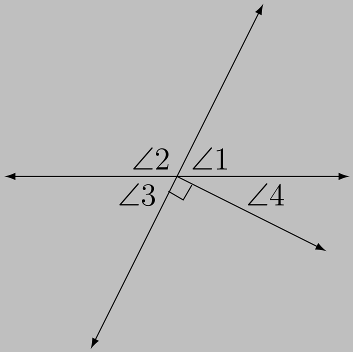
Pairs of Angles



Adjacent Angles - Two angles with a common side (ray).

Linear Pair - Two adjacent angles whose non common sides are opposite rays.

Pairs of Angles



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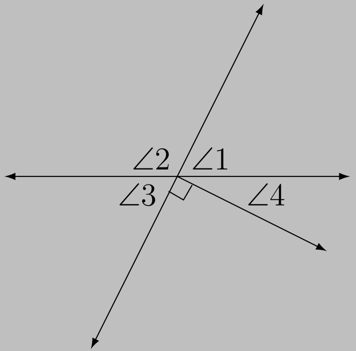

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Complementary Angles - Two angles whose measures sum to $90°$.

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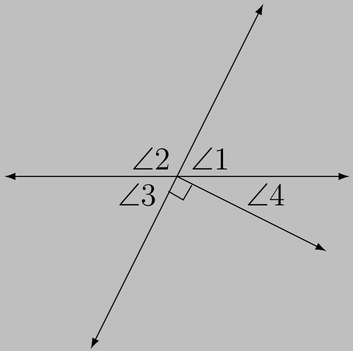

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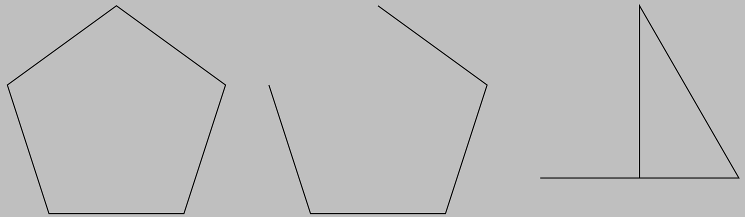
Supplementary Angles - Two angles whose measures sum to $180°$.

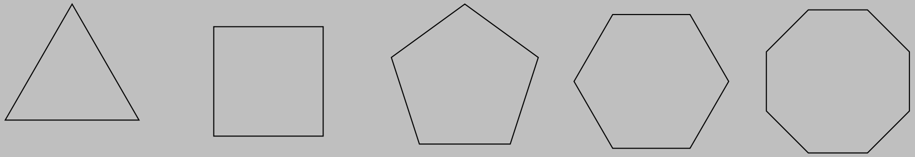
Polygons

A polygon is a closed figure within a single plane formed by 3 or more lines segments (sides), where no two sides with a common endpoint are collinear.



polygon   not closed   collinear sides

Named Polygons

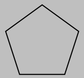


triangle quadrilateral pentagon hexagon octagon

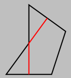
Sides	Name
7	Heptagon
9	Nonagon
10	Decagon
12	Duo decagon
n	n-gon

Types of Polygons

Convex polygon - No line formed by extending a side intersects any point interior to the polygon.



Concave polygon - At least one line formed by extending a side intersects points in the interior.



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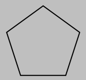

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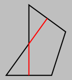
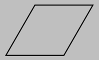


Equilateral polygon - All sides are congruent.



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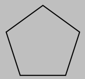

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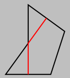

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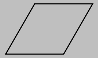

Equiangular polygon - All angles are congruent.

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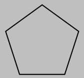

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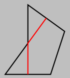

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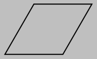

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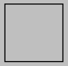
Regular polygon - Both equilateral and equiangular.



Congruent Polygons

Polygons are congruent if their corresponding sides are congruent and their corresponding angles are congruent.

$$\triangle ABC \cong \triangle DEF$$

if

$$\overline{AB} \cong \overline{DE} \quad \overline{BC} \cong \overline{EF} \quad \overline{CA} \cong \overline{FD}$$

and

$$\angle A \cong \angle D \quad \angle B \cong \angle E \quad \angle C \cong \angle F$$

Congruent Polygons

Polygons are congruent if their corresponding sides are congruent and their corresponding angles are congruent.

$$\triangle ABC \cong \triangle DEF$$

if

$$\overline{AB} \cong \overline{DE} \quad \overline{BC} \cong \overline{EF} \quad \overline{CA} \cong \overline{FD}$$

and

$$\angle A \cong \angle D \quad \angle B \cong \angle E \quad \angle C \cong \angle F$$

In other words:

Congruent figures have the same size and shape.

Congruent figures differ only by a translation, rotation or reflection.

Euclidean Postulates

Postulate 1 There exists a straight line segment between any two points.

Postulate 2 Any line segment can be extended to form a straight line.

Postulate 3 Given a line segment, one can draw a circle passing through one endpoint with the center at the other endpoint.

Postulate 4 All right angles are congruent.

Postulate 5 If two lines are drawn which intersect a third in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.